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A METHOD FOR THE CONSTRUCTION OF THE APPROXIMATE SOLUTION OF THE MIXED AXISYMMETRIC PROBLEM IN THE THEORY OF ELASTICITY

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In this work some general considerations are presented with respect to the construction of an approximate solution to spatial mixed problems in the theory of elasticity. The axisymmetric problem is used as an example.

For the solution a structure is proposed which permits to satisfy exactly mixed boundary conditions of a certain type. In addition, this structure contains a series of arbitrary functions the selection of which can be made such that the system of differential equations for the equilibrium of the elastic body is satisfied in the best possible manner (in one sense or another).

The analyses are based on the utilization of R-functions [1] which makes it possible to examine practically any real three-dimensional bodies. The question of the foundation of the method is not discussed.

1. Let us examine a system of functions $H\{\varphi_i(x_1,x_2)\}$ $(i = 1,2,\ldots,n)$ belonging to the class $C^{(k)}$. The system is called the H-system of base functions. We shall utilize this system in the subsequent construction of coordinate sequences. Just as in paper [2], the functions which can be constructed with the aid of this base system will be called H-realizable. The set of H-realizable functions are designated by the symbol $M(H)$.

For any function $f(x_1, x_2) \in M(H)$ it is possible to find in the plane x_1x_2 some corresponding figure L which is determined by the equation $f(x_1, x_2) = 0$. (The figure may turn out to be an empty set). The figure L is called an H-realizable figure. The set of H-realizable figures is designated by $N(H)$.

In the plane x_1x_2 the set of domains defined by an inequality of the form

$$f(x_1, x_2) \geqslant 0, \qquad f(x_1, x_2) \in M(H) \tag{1.1}$$

will be called the set of H-realizable domains and denoted by the symbol $G(H)$.

It is apparent that the sets $M(H)$, $N(H)$ and $G(H)$ are completely determined for a given base system of functions $H\{\varphi_i(x_1,x_2)\}$ $(i = 1,2,\ldots, n)$.

In papers [1, 2] the concept of algorithmic completeness of the system H of base functions is introduced and it is shown that if the system is algorithmically complete, then with the aid of this system we can write the equation for any figure.

If the following functions are taken as the system H

$$\begin{gathered}\varphi_1(x_1,x_2) = x_1 + x_2, \quad \varphi_2(x_1,x_2) = x_1x_2 \\ \varphi_3(x_1, x_2) = {}^1/_2\,(x_1 + x_2 - \sqrt{x_1^2 + x_2^2 - 2\alpha x_1x_2})\,(x_1^2 + x_2^2)^{1/2k} \\ \varphi_4(x_1, x_2) = {}^1/_2\,(x_1 + x_2 + \sqrt{x_1^2 + x_2^2 - 2\alpha x_1x_2})\,(x_1^2 + x_2^2)^{1/2k} \\ -1 < \alpha < 1; \quad \varphi_5(x_1,x_2) = \bar{x}_1 = -x_1, \quad \varphi_6(x_1x_2) \\ \varphi_7(x_1,x_2),\ldots, \varphi_n(x_1,x_2), \qquad \varphi_i(x_1, x_2) \in C^{(k)}\ (i = 6,7,\ldots, n)\end{gathered} \tag{1.2}$$

then the system turns out to be algorithmically complete in the class $C^{(k)}$. This makes it possible to construct the function $\omega(x_1,x_2) \in M(H)$ which becomes zero at points (and only at points) of any prescribed figure $L \in N(H)$. Numerous examples for the construction of the function $\omega(x_1,x_2)$ for closed and open curves are presented in papers [1–5].

In paper [5] a general algorithm is given for the construction of the function $\omega \in M(H)$ which satisfies the following conditions

$$\omega(x_1, x_2) = 0, \qquad d\omega/d\nu = 1, \quad (x_1, x_2) \in L \tag{1.3}$$

$$\omega(x_1, x_2) > 0, \quad \text{when } (x_1, x_2) \in (S) \tag{1.4}$$

where ν is the direction of the internal normal to the curve L. (We note that the second condition (1.3) has a meaning for points which are not corner points). By (S) we shall denote an H-realizable domain bounded by an H-realizable closed curve. In the case where the curve L is open, (S) will be taken as the part of the plane lying to the right or to the left of the curve. Sometimes it is conveneient to consider the figure L as the sum of figures $L_i (i = 1,2,\ldots, p)$. In this case only on L_i will the function $\omega(x_1, x_2) = 0$ be supplied with the index i.

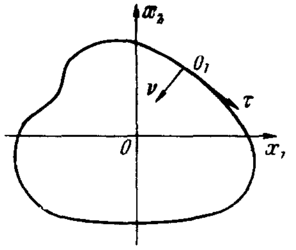

Fig. 1

On the curve L a system of coordinates (ν, O_1, τ) is selected so that for an observer looking along the direction $O_1\nu$ the axis $O_1\tau$ will be pointing to the left (Fig. 1).

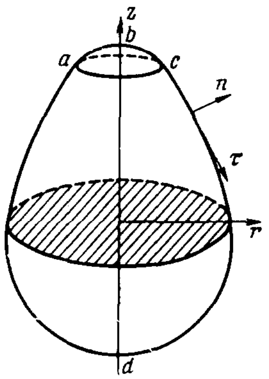

Fig. 2

Let us introduce the operators of differentiation

$$D_1 = \frac{\partial\omega}{\partial x_1}\frac{\partial}{\partial x_1} + \frac{\partial\omega}{\partial x_2}\frac{\partial}{\partial x_2} \tag{1.5}$$

$$T_1 = \frac{\partial\omega}{\partial x_1}\frac{\partial}{\partial x_2} - \frac{\partial\omega}{\partial x_2}\frac{\partial}{\partial x_1} \tag{1.6}$$

It is not difficult to establish that they have the following properties

$$D_1(u)_{|L} = \partial u/\partial\nu, \qquad D_1(\omega)_{|L} = 1 \tag{1.7}$$

$$T_1(u)_{|L} = du/d\tau, \quad T_1(\omega) \equiv 0 \tag{1.8}$$

In fact, by virtue of the second condition (1. 3) we have

$$\frac{d\omega}{dx_1}_{|L} = [\,|\operatorname{grad}\omega|\cos(\nu, x_1)]_L = \frac{\partial\omega}{\partial\nu}\cos(\nu, x_1) = \cos(\nu, x_1) \tag{1.9}$$

(| grad ω | on L is equal to $d\omega/d\nu$, since $\omega = 0$ is one of the level curves of function $\omega(x_1, x_2)$). By analogy we find that

$$\frac{\partial\omega}{\partial x_2}_{|L} = \cos(\nu, x_2) \tag{1.10}$$

Consequently,

$$D_1(u)\,|_L = \frac{\partial u}{\partial x_1}\cos(\nu, x_1) + \frac{\partial u}{\partial x_2}\cos(\nu, x_2) = \frac{\partial u}{\partial\nu}$$

$$T_1(u)\,|_L = \frac{\partial u}{\partial x_2}\cos(\nu, x_1) - \frac{\partial u}{\partial x_1}\cos(\nu, x_2) = \frac{\partial u}{\partial\tau}$$

The second equalities in formulas (1. 7) and (1. 8) are obvious. Operators D_1 and T_1 are linear $D_1(u+v) = D_1(u) + D_1(v),\ T_1(u+v) = T_1(u) + T_1(v)$

and it is easy to verify that the formulas for product differentiation are applicable to these operators $D_1(uv) = D_1(u)v + uD_1(v),\quad T_1(uv) = T_1(u)\,v + uT_1(v)$

Frequent use will be made of these operators.

2. In the cylindrical system of coordinates (r, φ, z) let us examine the axisymmetric problem of the theory of elasticity for a body which is obtained by revolution of the H-realizable curve L around the axis oz (Fig. 2) under the conditions

$$u_z(r,z) = u^\circ(r, z) \quad \text{on} \quad (S_1) \tag{2.1}$$

$$\sigma_n(r,z) = \sigma_n^\circ(r,z) \quad \text{on} \quad (S_2) \tag{2.2}$$

$$\tau_n(r,z) = \tau_n^\circ(r, z) \quad \text{on} \quad (S) \tag{2.3}$$

where u_z is the displacement along the z-axis, and τ_n and σ_n are the tangential and the normal stresses.

We set $u_1 = u_r$ and $u_2 = u_z$, in addition to this let (S) be the surface bounding the body of revolution (V) ; (S_1) and (S_2) are the parts (abc) and (cda) of this surface, **n** is the direction of the external normal. With respect to the given functions u°, σ_n° and τ_n° it will be assumed that the first one is continuous and the other two are piecewise continuous. Boundary conditions of this type are applicable for example to contact problems [6].

The problem will consist in finding such a structure of functions u_1 and u_2 which will satisfy the boundary conditions (2.1) and (2.3). Furthermore, this structure should have a certain degree of freedom so that within the framework of this structure of solution it will be possible to approach to any degree the functions from a class of functions which satisfy mixed boundary conditions (2.1) and (2.3).

The boundary conditions (2.2) and (2.3) are written in the expanded form

$$(\lambda+2\mu)\left[\frac{\partial u_1}{\partial n}\cos(n,r)+\frac{\partial u_2}{\partial n}\cos(n,z)\right]+\lambda\left[\frac{\partial u_2}{\partial \tau}\cos(n,r)-\frac{\partial u_1}{\partial \tau}\cos(n,z)+\frac{u_1}{r}\right]= \\ =\sigma_n{}^\circ(r,z) \text{ on } (S_2) \tag{2.4}$$

$$\mu\left[\frac{\partial u_1}{\partial n}\cos(n,z)-\frac{\partial u_2}{\partial n}\cos(n,r)+\frac{\partial u_1}{\partial \tau}\cos(n,r)+\frac{\partial u_2}{\partial \tau}\cos(n,z)\right]=\tau_n{}^\circ(r,z) \text{ on } (S) \tag{2.5}$$

Conditions (2.4) and (2.5) written above are extended in a continuous manner into region (V) by means of operators (1.5), (1.6) and Eqs. (1.9), (1.10)

$$(\lambda+2\mu)\left[D_{11}(u_1)\frac{\partial\omega}{\partial r}+D_{11}(u_2)\frac{\partial\omega}{\partial z}\right]+\lambda\left[T_1(u_2)\frac{\partial\omega}{\partial r}-T_1(u_1)\frac{\partial\omega}{\partial z}+\frac{u_1}{r}\right]= \\ =F_1(r,z)+\omega_2\varphi_{10}(r,z) \tag{2.6}$$

$$\mu\left[D_1(u_1)\frac{\partial\omega}{\partial z}-D_1(u_2)\frac{\partial\omega}{\partial r}-T_1(u_1)\frac{\partial\omega}{\partial r}-T_1(u_2)\frac{\partial\omega}{\partial z}\right]=F_2(r,z)+\omega\varphi_{20}(r,z) \tag{2.7}$$

Here

$$D_{11}=\left(1+\frac{\omega_2}{\omega_1+\omega_2}\right)D_1=\begin{cases}2D_1 & \text{on } (S_1)\\ D_1 & \text{on } (S_2)\end{cases} \tag{2.8}$$

$$\omega(r,z)=0,\quad \partial\omega/\partial\nu=1,\quad \text{when}\quad (r,z)\in(S)$$

$$\omega_i(r,z)=0,\quad \partial\omega_i/\partial\nu=1,\quad \text{when}\quad (r,z)\in(S_i)\quad (i=1,2)$$

Functions ω and ω_i are strictly positive in the domain (V); φ_{10} and φ_{20} are so far completely arbitrary functions. The functions F_1 and F_2 realize the continuous extension of functions $\sigma_n{}^\circ$ and $\tau_n{}^\circ$ into the domain (V) and consequently have the properties

$$F_1=\sigma_n{}^\circ \text{ on } (S_2),\quad F_2=\tau_n{}^\circ \text{ on } (S) \tag{2.9}$$

In contrast to (2.4) and (2.5), the relationships (2.6) and (2.7) are valid everywhere inside the domain (V). By virtue of relationships (1.7), (1.10) and (2.9) they transform on the boundary of the domain into boundary conditions (2.4) and (2.5).

3. We have the solution of the problem in the form

$$u_1=\psi_{11}+\omega\psi_{12},\qquad u_2=\psi_{21}+\omega\psi_{22} \tag{3.1}$$

where ψ_{ij} are some functions with respect to which we assume that they are no less than twice continuously differentiable in the domain (V).

In order to satisfy the first of the boundary conditions of the problem, it is sufficient to assume

$$\psi_{21}=f(r,z)+\omega_1\psi_{23}$$

where $f(r,z)$ is a function in the domain (V) which can be differentiated continuously the required number of times and which satisfies the condition

$$f(r,z)=u^\circ(r,z),\qquad \text{when } (r,z)\in(S_1)$$

Substituting functions (3.1) into relationships (2.6) and (2.7), taking into account the properties of operators D_1 and T_1, we find

$$(\lambda + 2\mu)\left\{[D_{11}(\psi_{12})\,\omega + D_{11}(\omega)\,\psi_{12}]\frac{\partial\omega}{\partial r} + [D_{11}(\omega)\,\psi_{22} + \omega D_{11}(\psi_{22})]\frac{\partial\omega}{\partial z}\right\} +$$
$$+ \lambda\left\{[T_1(\omega)\,\psi_{22} + \omega T_1(\psi_{22})]\frac{\partial\omega}{\partial r} - [T_1(\omega)\,\psi_{12} + \omega T_1(\psi_{12})]\frac{\partial\omega}{\partial z} + \frac{\lambda}{r}\,\omega\psi_{12}\right\} =$$
$$= \Phi_1 + \omega_2\varphi_{10} \tag{3.2}$$

$$\mu\left\{[D_1(\omega)\,\psi_{12} + \omega D_1(\psi_{12})]\frac{\partial\omega}{\partial z} - [D_1(\omega)\,\psi_{22} + \omega D_1(\psi_{22})]\frac{\partial\omega}{\partial r} -\right.$$
$$\left. - [T_1(\omega)\,\psi_{12} + \omega T_1(\psi_{12})]\frac{\partial\omega}{\partial r} - [T_1(\omega)\,\psi_{22} + \omega T_1(\psi_{22})]\frac{\partial\omega}{\partial z}\right\} = \Phi_2 + \omega\varphi_{20} \tag{3.3}$$

where

$$\Phi_1 = F_1 - (\lambda + 2\mu)\left[D_{11}(\psi_{11})\frac{\partial\omega}{\partial r} + D_{11}(\psi_{21})\frac{\partial\omega}{\partial z}\right] -$$
$$- \lambda\left[T_1(\psi_{21})\frac{\partial\omega}{\partial r} - T_1(\psi_{11})\frac{\partial\omega}{\partial z} + \frac{\lambda}{r}\psi_{11}\right] \tag{3.4}$$

$$\Phi_2 = F_2 - \mu\left[D_1(\psi_{11})\frac{\partial\omega}{\partial z} - D_1(\psi_{21})\frac{\partial\omega}{\partial r} - T_1(\psi_{11})\frac{\partial\omega}{\partial r} - T_1(\psi_{21})\frac{\partial\omega}{\partial z}\right] \tag{3.5}$$

Since

$$\left.\frac{\partial\omega_i}{\partial\nu}\right|_{S_i} = 1$$

the following equation applies in the domain (V) :

$$D_{11}(\omega) = 1 + \omega\chi_0 = 1 + \omega_2\chi_1 \tag{3.6}$$

where χ_0 and χ_1 are known functions.

If we take advantage of Eq. (2.6) and the arbitrariness of functions φ_{10} and φ_{20}, then relationships (3.2) and (3.3) can be written in the form of a system of equations for functions ψ_{12} and ψ_{22}

$$(\lambda + 2\mu)\left(\psi_{12}\frac{\partial\omega}{\partial r} + \psi_{22}\frac{\partial\omega}{\partial z}\right) = \Phi_1 + \omega_2\varphi_{11}$$

$$\mu\left(\psi_{12}\frac{\partial\omega}{\partial z} - \psi_{22}\frac{\partial\omega}{\partial r}\right) = \Phi_2 + \omega\varphi_{21} \tag{3.7}$$

where φ_{11} and φ_{21} are new arbitrary functions which were obtained as a result of combining terms with factors ω and ω_2.

The determinant of this system

$$\Delta = \mu(\lambda + 2\mu)\left[\left(\frac{\partial\omega}{\partial r}\right)^2 + \left(\frac{\partial\omega}{\partial z}\right)^2\right] = \mu(\lambda + 2\mu)\,|\operatorname{grad}\omega|^2$$

is a function of (r, z) which in the domain (V) is different from zero everywhere with the exception of points of the extremum and the saddle points of the function ω.

The formal solution of this system has the form

$$\psi_{12} = \frac{1}{\Delta}\left[(\lambda + 2\mu)(\Phi_2 + \omega\varphi_{21})\frac{\partial\omega}{\partial z} + \mu(\Phi_1 + \omega_2\varphi_{11})\frac{\partial\omega}{\partial r}\right] \tag{3.8}$$

$$\varphi_{22} = \frac{1}{\Delta}\left[\mu(\Phi_1 + \omega_2\varphi_{11})\frac{\partial\omega}{\partial z} - (\lambda + 2\mu)(\Phi_2 + \omega\varphi_{21})\frac{\partial\omega}{\partial r}\right] \tag{3.9}$$

It is easy to verify that the formal solution (3.8),(3.9) also applies at those points at which $\Delta = 0$. In fact, since points at which $\Delta = 0$ lie inside the region (V) and since at these points ω and ω_2 are different from zero, it is sufficient to select the arbitrary functions φ_{11} and φ_{21} in the form

$$\varphi_{11} = \frac{1}{\omega_2 (r_0, z_0)} \left[-\Phi_1 + \Delta (\Phi_1 + \omega_2 \varphi_{31})\right] \tag{3.10}$$

$$\varphi_{21} = \frac{1}{\omega (r_0, z_0)} \left[-\Phi_2 + \Delta (\Phi_2 + \omega \varphi_{32})\right]$$

where φ_{31} and φ_{32} are new arbitrary functions, $A(r_0, z_0)$ is the point at which $\Delta = 0$.

We note that for such a selection of functions φ_{11} and φ_{21} the formal solution of system (3.7) retains its form at those points also at which $\Delta = 0$. If there are n such points, then it will be necessary to subject functions φ_{11} and φ_{21} to n conditions of the type (3.10), while the form of the solution remains the same. However, in practice it will not be necessary to do that, because solution (3.8), (3.9) can be substantially simplified if we take into account that

$$\Delta = \mu (\lambda + 2\mu) + \omega \chi_2 \quad \text{or} \quad \frac{1}{\Delta} = \frac{1}{\mu (\lambda + 2\mu)} + \omega \chi_3,$$

where χ_2 and χ_3 are known functions.

Rearranging terms which contain ω and ω_2 in (3.8), (3.9) we find

$$\psi_{12} = \frac{1}{\mu (\lambda + 2\mu)} \left[(\lambda + 2\mu) \Phi_2 \frac{\partial \omega}{\partial z} + \mu \Phi_1 \frac{\partial \omega}{\partial r}\right] + \omega_2 \varphi_{33} \tag{3.11}$$

$$\psi_{22} = \frac{1}{\mu (\lambda + 2\mu)} \left[\mu \Phi_1 \frac{\partial \omega}{\partial z} - (\lambda + 2\mu) \Phi_2 \frac{\partial \omega}{\partial r}\right] + \omega_2 \varphi_{34} \tag{3.12}$$

where φ_{33} and φ_{34} are arbitrary functions as before.

The solution of system (3.7) written in the form (3.11), (3.12) now does not contain the function Δ in the denominator (the function Δ is eliminated through an appropriate selection of arbitrary functions which enter into the initial form of the solution) and has a meaning everywhere in the domain (V).

The functions u_1 and u_2 depend on two arbitrary fundamental functions ψ_{11} and ψ_{23} and two arbitrary auxiliary functions φ_{33} and φ_{34} In this case all boundary conditions of the problem will be satisfied. The arbitrariness of functions ψ_{ij} will be utilized in satisfying Lame's system of equations.

If functions ψ_{ij} which enter into the structure of functions u_1 and u_2 are expanded in series with respect to some complete orthonormalized system of functions and if a finite number of terms is retained in the expansions, then two sequences of functions $u_1^{(k)}$ and $u_2^{(k)}$ are obtained which satisfy all conditions of the mixed problem.

Leaving aside for a while important questions with regard to the proof of completeness of sequences $u_1^{(k)}$ and $u_2^{(k)}$, let us just point out that the proposed structure of the solution has some properties of complete systems.

For this purpose we write the equation of distribution of normal stress on the region (S_1)

$$\sigma_n \big|_{S_1} = F_1 - (\lambda + 2\mu) \left[\left(\frac{\partial \omega}{\partial r}\right)^2 \frac{\partial \psi_{11}}{\partial r} + \frac{\partial \omega}{\partial r} \frac{\partial \omega}{\partial z} \left(\frac{\partial \psi_{11}}{\partial z} + \frac{\partial f}{\partial r}\right) + \right.$$
$$\left. + \left(\frac{\partial \omega}{\partial z}\right)^2 \frac{\partial f}{\partial z} + \frac{\partial \omega}{\partial z} \psi_{23}\right] + \omega_2 \varphi_{35} \tag{3.13}$$

here φ_{35} is some function.

In the last equation functions ψ_{11} and ψ_{23} are represented in the following form:

$$\psi_{11} = \psi_{13} + \omega \frac{\partial \omega}{\partial r} \psi_{14}, \qquad \psi_{23} = \frac{\partial \omega}{\partial z} \psi_{14} + \psi_{15}$$

where ψ_{ij} are some new functions. Elementary transformations lead to the relationship

$$\sigma_n \big|_{S_1} - F_1 - \omega_2 \psi_{35} + (\lambda + 2\mu) \left[\left(\frac{\partial \omega}{\partial r}\right)^2 \frac{\partial \psi_{13}}{\partial r} + \right. \tag{3.14}$$

$$+\frac{\partial\omega}{\partial r}\frac{\partial\omega}{\partial z}\left(\frac{\partial\psi_{13}}{\partial z}+\frac{\partial f}{\partial r}\right)+\left(\frac{\partial\omega}{\partial z}\right)^2\frac{\partial f}{\partial z}+\frac{\partial\omega}{\partial z}\psi_{15}\Big]=-(\lambda+2\mu)\psi_{14}$$

From the last equation it is clear that the arbitrariness of function ψ_{14}, and consequently of functions ψ_{11} and ψ_{23}, is quite sufficient in order to ensure the necessary values of the normal stress σ_n on the region (S_1).

Note. The axial symmetry of the problem is preserved if the condition (2.1) is replaced by the condition replaced by the condition

$$u_1(r,z) = u^\circ(r,z) \quad \text{on} \quad (S_1) \tag{3.15}$$

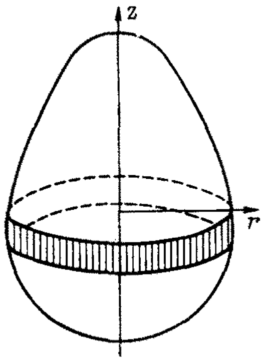

Fig. 3

where (S_1) is the region of (S) shaded in Fig. 3. For this case everything presented above retains its validity, only function f should be set identically equal to zero and function ψ_{11} should be selected in the form

$$\psi_{11} = f_1(r,z) + \omega_1\psi_{13}^* \tag{3.16}$$

where $f_1(r,z) = u^\circ(r,z)$, when $(r,z) \in (S_1)$.

It should also be noted that since the extension of function $u^\circ(r, z)$ into the domain (V) can be accomplished by many methods, it is somehow necessary to utilize this freedom in a reasonable way. It is possible for example to accomplish the extension in such a manner that the derivatives of the function $f(r, z)$ will have the same singularities as the function which is sought at corner points or at points of boundary condition separation. In this manner it is possible to introduce into the approximate solution some fundamental features of the exact solution. This apparently decreases the "loading" on the function ψ_{ij}.

Sometimes it is possible to take as the function $f(r, z)$ the exact solution of a problem which is close to the problem under investigation. For example, in the problem of a cylinder of finite height which is compressed at the ends by rigid punches it is possible to take as function $f(r,z)$ the solution of the problem for a layer which is compressed by two punches of the same kind.

In the second part of this paper examples of solutions will be presented for some concrete problems with computations performed on a digital computer.

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